

# Philosophy 0540

## Deductive Logic

Course Site: <http://frege.org/phil0540/>

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### What Is Deductive Logic?

Logic is the study of what makes an inference, in a certain limited sense, ‘good’, ‘valid’, or ‘correct’. Logic, as the great logician (and founder of modern logic) Gottlob Frege convincingly argued, is not a branch of psychology: It does not concern itself with how people do in fact reason, with what sorts of arguments they find compelling, nor even with whether a given argument in fact shows its conclusion to be true. Logic is, instead, a normative discipline: It is about one important constraint on what it is to reason or argue correctly. Logic is concerned with how people ought to reason, that is, with what rules they ought to follow when they do reason; it concerns itself with whether, if one accepts the assumptions someone is making, one must also (on pain of irrationality) either accept the conclusion for which she is arguing or else give up one of those assumptions.

One should not, however, expect this to be a course in reasoning or argument. Logic studies the principles of valid argument abstractly: While the course should teach you something about distinguishing valid from invalid arguments—and, like any good course, should teach you something beyond its specific subject-matter, something which will help you with other courses (and in your life after all the courses are over)—this course is not designed to help you write or reason better. What the course will do is introduce you to the fundamental concepts of modern mathematical logic.

We shall seek to characterize valid arguments of two different types. In order to do so, however, we shall have to introduce a great deal of special symbolism: We wish to consider,

not specific arguments, but kinds of arguments; and we want to see, for example, what is common to the good, or ‘valid’, arguments “John is at home; so either he is at home or at the zoo” and “Tom is a professor; so he is either a professor or a fireman”.

As part of our study of logic, we will develop a ‘formal system’ in which to prove that various arguments are, indeed, valid. Much of this middle part of the course will be something like a high school geometry class, as we shall be learning to do proofs in this system, just as one learns, in high school, to do proofs in axiomatic geometry.

Finally, we shall turn our attention upon the formal system itself and study it. We shall ask such questions as: Is it possible to prove, in this system, that any given valid argument really is valid? Or are there some valid arguments whose validity can not be demonstrated in this system? Is there some kind of way to decide or to calculate whether an argument is valid?

## Course Structure and Requirements

The course will meet Monday, Wednesday, and Friday at 10am, in Smith-Buonanno 106. Class meetings will consist primarily of lectures. The course website will always contain the most up-to-date information about it. It can be found at <http://frege.org/phil0540/>.

The text for the course is *Deductive Logic*, by Warren Goldfarb. List price is \$39.00. Copies are available at the Brown bookstore. Students should plan to read the relevant material from the book before each lecture. Lectures will not cover all material for which students will be responsible.

There are three open ‘Q&A’ sessions held each week:

- Monday, 6-7pm, location TBA (led by Larry Kenny)
- Wednesday, 3-4pm, locatio TBA (led by Mark Christman)
- Thursday, 10:30-11:30am, location TBA (led by Rachel Leadon)

Please note that these will not start until rooms are assigned, which may take a bit.

There will be a mid-term examination on 13 October and a final examination on 19 December, at 9am. There will also be seven problem sets. Please see below for policies on grading.

## Electronics in Class

Students should not use any electronic devices in class, including laptops, tablets, and mobile phones. (Digital paper is fine.) Research has shown that using such devices not only adversely affects one’s own performance—students who take notes on a computer process the information less completely than those who take notes by hand—but that it also adversely affects the performance of other students whom it distracts. (See here and here, for example.)

Students who for some reason *need* to use electronic devices in class should speak to the instructor.

## Prerequisites

There are no formal prerequisites for this course. In particular, the course presupposes no college-level mathematical knowledge. However, much of the course is mathematical in content: Some familiarity, experience, and comfort with proofs, such as those in a high-school geometry course, is extremely useful. Anyone uncertain of their background in this area is encouraged to speak with the instructor.

## How To Learn Logic

There are two crucial points to keep in mind.

- This course is fairly easy at the beginning, but it starts to get more difficult after the mid-term, and it then quickly becomes *very* difficult. And the course is very cumulative: What we do later always depends heavily on what we've done earlier. If you get behind, it can be very difficult to catch up. It is therefore *absolutely impossible* to learn this material in the two weeks before the final exam, and, if you try to learn it that way, I can pretty much guarantee that you will fail the course. People do fail the course each time it is offered for this very reason. Don't be one of them.
- As with any mathematical subject-matter, it is impossible to learn this material without doing a lot of exercises. The book contains many more than are assigned, and students are encouraged to do additional exercises to improve their understanding of the material. Students are also encouraged to work on the problems together—though, of course, submitted material should be a student's own work.

That said, here is how to learn logic:

- You should plan to attend the lectures. I will present the material in a way somewhat different from how it is presented in the book. Moreover, not all material for which students are responsible is in the book.
- You should expect to spend about an hour and a half per class reviewing the material from that lecture and then reading the material for the next lecture. When we have review sessions (which we have for every problem set except the first), you should plan to spend this time reviewing the material for that problem set. Figure out what questions you have beforehand, so you can ask them in class.
- You should plan to attend at least one of the Q&A sessions each week.
- As I have already said, *learning* logic means *doing* logic, so doing the problem sets is perhaps the most important thing you will do. Note that the early problem sets are easier, and you should expect them to take less time. As the class progresses, the problem sets become harder, and you should expect those to take more time.
- If you have any difficulties while doing the problem sets, or do not understand something that was said in lecture, then contact us for help right away. We are here to help you learn! You can visit us at office hours, or send us email, or just talk to us after class. You can ask any one of us for help. You do not specially need to visit the person who happens to be grading your problem sets.

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You should thus expect to spend about 181 hours of time on this class, which breaks down as follows:

- Class time: 40 hours
- Reading and reviewing: 60 hours
- Q&A sessions: 12 hours
- Problem sets: 6 hours, on average, per problem set, totaling 42 hours
- Reviewing for the mid-term exam: 8 hours
- Reviewing for the final exam, including time spent at the formal review sessions: 16 hours
- The final exam: 3 hours

## Grading Policies

Performance on the mid-term examination, held in class on 13 October, the final examination, held on 19 December, at 9:00am, and the seven problem sets will contribute to determining a student's grade for the course. The grade itself will be determined by a variety of factors.

- The first and most important factor is that *all of the problem sets must be completed and submitted for marking*. Failure to submit all of the problem sets will *automatically* lead to a grade of NC. It is, quite simply, impossible to learn this material without doing a lot of problems, and students should actually plan to do a lot more problems than are actually assigned.
  - OK, actually, we'll let you off once, if you do miss a problem set. But you will get **no credit** for that problem set, which will **badly affect** your overall score and have other consequences listed below. This should **not** be thought of as a 'freebie'.
  - Please note that the requirement is that the problem sets should be "completed", and by that I mean that one has given them a proper effort. Simply turning in a piece of paper with a few random jottings does not count as completing a problem set.
- A presumptive grade will be determined by performance on the two exams, with twice as much weight being given to the final. Borderline cases will be decided by performance on the problem sets (and if you fail to turn in all the problem sets, you will automatically receive the lower grade). Exceptionally good or bad performance on the problem sets may move a grade up or down.
- *If you turn in all the problem sets*, then it is impossible to do worse in this class than you do on the final exam. That is: If you get an A on the final (and have turned in all the problem sets), you will get an A for the course; if you get a B on the final, you cannot get worse than a B for the course, though you might get an A if your mid-term and problem sets were good enough.

- Effort matters a lot. It is *impossible* to fail this class if you have given it what we regard as proper effort. That would mean such things as coming for help, if you need it, not to mention turning in all the problem sets.

Problem sets are due in class on the day specified below. *We will not accept late problem sets*, as late sets make the graders' task much more difficult. On the other hand, we are quite prepared to grant extensions, so long as they are requested in advance, that is, *by no later than 5pm the day before the problem set is due*. (Such requests should be directed by email to the instructor, cc'ing your grader.) Extensions will not be granted after that time except in very unusual and unfortunate circumstances.

Because we are so reasonable, exploitation of our reasonableness will be taken badly. Do not make a habit of asking for extensions. We will grant one, and maybe another, but that is about as far as we are prepared to go, unless you have some very good reason.

The grading scale for the problem sets can be found below, together with the assigned problems for each set.

Let me emphasize again something said above. As with any mathematical subject-matter, it is impossible to learn this material without doing a lot of exercises. The book contains many more exercises than are assigned, and students are encouraged to do additional exercises to improve their understanding of the material. Students are also encouraged to work on the problems together—though, of course, submitted material should be a student's own work.

## Syllabus

|              |                      |
|--------------|----------------------|
| 6 September  | Introductory Meeting |
| 8 September  | Sections 2–5, 7      |
| 11 September | Sections 6, 8        |

End of material covered by Problem Set #1: Due 18 September

|              |                    |
|--------------|--------------------|
| 13 September | Section 9          |
| 15 September | Sections 10–11, 13 |
| 18 September | Section 14         |
| 20 September | Sections 14–15     |

End of material covered by Problem Set #2: Due 29 September

|              |                                       |
|--------------|---------------------------------------|
| 22 September | Review Session                        |
| 22 September | Section 16                            |
| 27 September | Introduction to Quantification Theory |
| 29 September | Sections 18–19                        |
| 2 October    | Sections 20–22                        |

End of material covered by Mid-term Examination

|           |            |
|-----------|------------|
| 4 October | Section 23 |
| 6 October | Section 24 |

End of material covered by Problem Set #3: Due 18 October

|            |  |
|------------|--|
| 9 October  | No Class: Indigenous Peoples' Day              |
| 11 October | Review Session                                 |
| 13 October | Mid-Term Examination                           |
| 16 October | Section 27                                     |
| 18 October | Introduction to Polyadic Quantification Theory |
| 20 October | Sections 28–9                                  |
| 21 October | Section 29                                     |
| 23 October | Section 29                                     |
| 25 October | Section 29                                     |

End of material covered by Problem Set #4: Due 1 November

|            |                |
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| 27 October | Review Session |
| 30 October | Section 30     |

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|------------|----------------|
| 1 November | Sections 30–31 |
| 3 November | Review Session |
| 6 November | Sections 31–32 |

End of material covered by Problem Set #5: Due 13 November

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|-------------|----------------|
| 8 November  | Section 33     |
| 10 November | Section 33     |
| 13 November | Section 33     |
| 15 November | Sections 33–34 |
| 17 November | Sections 33–34 |

End of material covered by Problem Set #6: Due 27 November

|             |                                |
|-------------|--------------------------------|
| 20 November | Section 41                     |
| 22 November | Review Session                 |
| 24 November | No Class: Thanksgiving Holiday |
| 27 November | Section 41                     |
| 29 November | Identity and Number            |
| 1 December  | Section 35                     |
| 4 December  | Section 35                     |
| 6 December  | Section 37                     |

End of material covered on Problem Set #7: Due 12 December

|             |                               |
|-------------|-------------------------------|
| 8 December  | Review Session                |
| TBA         | Review Session for Final Exam |
| 19 December | Final Exam, 9am               |

## Problem Sets

### Some Reminders

It is a requirement of the class that all of the problem sets must be completed and submitted for marking. (We'll let you off once, if you do miss one, but you will get *no credit* for that problem set.) Failure to submit all (but one) of the problem sets will automatically lead to a grade of NC. Please note that the requirement is that the problem sets should be “completed”, and by that I mean that one has given them a proper effort. Simply turning in a piece of paper with a few random jottings does not count as completing a problem set.

As with any mathematical subject-matter, it is impossible to learn this material without doing a lot of exercises. The book contains many more exercises than are assigned, and students are encouraged to do additional exercises to improve their understanding of the material. Students are also encouraged to work on the problems together—though, of course, submitted material should be a student's own work.

|               |   |
|---------------|---|
| Problem Set 1 | Section IA (pp. 253-5): 1b,d; 2, 3; 4b,d,f,i,k  |
| Problem Set 2 | Section IB (pp. 255-60): 1a; 2a,c; 3a,c,e; 4b,d,f; 6; 7a,c<br>Note: Problems 13 and 14 are examples of the sorts of 'logic puzzles' that are found on the LSAT. They can be done by schematizing the various assertions and doing a truth-table, but it is probably easier just to think through them.<br>Section IC (pp. 260-4): 1; 2; 5; 6; Extra Problems 1–2<br>Problem 6 is quite difficult to do completely correctly. If you're struggling with it, then consider some examples, and see if you can find the pattern that lies behind this fact.<br>If you'd like an extra challenge, then try 7, 8, and 9.  |
| Problem Set 3 | Section IIA (pp. 265-7): 1b,d; 2b,d; 3b,d; 4a,d,g<br>You should <i>definitely</i> do more of these for practice, but you only need to turn those ones in (though you can also turn in more if you want us to look at them).<br>Section IIB (pp. 267-71): 1a,c; 3b,c,e; 5a,c; 6<br>For 6, you may use the “method of §25”, if you wish, though we will not discuss it. You probably should just give an informal argument (i.e., one in words).  |
| Problem Set 4 | Section IIIA (pp. 273-6): 1a,d,g; 2a,c; 3a,c; 5a,b; Extra Problem 3   |
| Problem Set 5 | Section IIIB (pp. 276-81): 1b,d; 2b,d,f   |
| Problem Set 6 | Section IIIB (pp. 276-81): 4a,b; 5; 6; 9; 14; Extra Problem 4<br>Note: In 4a, the question is asking you to specify predicates of <i>English</i> that have the properties mentioned. For example, suppose we were asked for a predicate that is irreflexive, symmetric, and intransitive. Then “(1)+(2) is odd” would do. This is irreflexive [ $\forall x \neg(x + x \text{ is odd})$ ], symmetric [ $\forall x \forall y (x + y \text{ is odd} \rightarrow y + x \text{ is odd})$ ], and intransitive [ $\forall x \forall y \forall z (x + y \text{ is odd} \wedge y + z \text{ is odd} \rightarrow \neg(x + z \text{ is odd}))$ ]. The first two of these are obvious. For the last, if the antecedent holds, then either $y$ is even, and $x$ and $z$ are both odd, or $y$ is odd, and $x$ and $z$ are both even. Either way, $x + z$ is even. |

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Problem Set 7      Section IV (pp. 284-8): 1b; 2b; 3b; 4b,d; 5a; 7  
Section IIIC (pp. 281-3): 1; 3; 6a  
Note: For problem (1), you need only do a deduction for one version  
of the CQ rule; you can choose which to do.

## Grading

Problem sets are graded on the following scale:

- 5 Better than perfect**  
To receive a 5, a problem set must not only get all the problems right (save possibly for a couple silly mistakes), but either do a number of the problems in a particularly elegant way, or else do some of the optional problems well.
- 4 More or less perfect**  
To receive a 4, a problem set must get nearly all the problems right (save possibly for some silly mistakes). This should be thought of as meaning that the problem set demonstrates a understanding of the material that is better than adequate as a basis for what we shall do henceforth.
- 3 Satisfactory**  
To receive a 3, a problem set must get most of the problems right, especially those that are “foundational”, i.e., those that test understanding of the basic concepts covered in that part of the course. This should be thought of as meaning that the problem set demonstrates a understanding of the material that is adequate as a basis for what we shall do henceforth and hence that gives us no reason for concern.
- 2 Unsatisfactory**  
A problem set that receives a 2 is one that reveals some misunderstanding of, or confusion about, the basic concepts covered in that section. Since the course is cumulative, it therefore gives us some concern about the student’s ability to absorb the material in later parts of the course. Students who receive a 2 are encouraged to seek help, as soon as possible, to try to resolve whatever problems were apparent from the problem set.
- 1 Worrying**  
A problem set will only receive a grade of 1 if it reveals a significant level of misunderstanding of, or confusion about, the basic concepts covered in that section. Since the course is cumulative, it therefore gives us serious concern about whether the student will be able to absorb the material in later parts of the course. Students who receive a 1 really *must* get in touch with one of the instructors as soon as possible, either by attending office hours or by making an appointment. Such students might also consider hiring a tutor.

Do not compare problem set scores across graders. Different graders always have slightly different standards, and there is not really anything to do about that other than to take it into account. Which we will.

## Extra Problems

### For Problem Set 2

1. There is an obvious sense in which  $p \supset q$  can be ‘defined’ as:  $\neg p \vee q$ . These two schemata *must* have the same truth-value. Now let ‘•’ be a symbol for exclusive ‘or’. Can you write a sentence, containing only ‘A’, ‘•’, and ‘ $\vee$ ’ (and parentheses) that defines ‘ $\neg A$ ’? How about if you use ‘•’, ‘ $\vee$ ’, and ‘ $\wedge$ ’? What about ‘•’ and ‘ $\supset$ ’?
2. Show that the following, called the DeMorgan Laws, are valid:
  - a)  $\neg(p \wedge q) \equiv \neg p \vee \neg q$
  - b)  $\neg(p \vee q) \equiv \neg p \wedge \neg q$
 Commit these to memory.

### For Problem Set 4

3. With the universe of discourse and vocabulary as in problem 2, in the book, translate the following into clear, idiomatic English:
  - a)  $\exists x[Sx \wedge \forall y(Ty \wedge \forall z(Sz \rightarrow Ryz) \rightarrow Lxy)]$
  - b)  $\forall x[Tx \wedge \exists y(Sy \wedge Ryx) \rightarrow \exists y(Ty \wedge \exists z(Sz \wedge \neg Rzy) \wedge Lyx)]$
  - c)  $\neg \exists x[Sx \wedge \forall y(Ty \rightarrow Lxy) \wedge \exists y(Ryx)]$

### For Problem Set 6

4. In each pair, deduce each from the other:
 

|                              |                           |
|------------------------------|---------------------------|
| a) $p \vee \exists x(Fx)$    | $\exists x(p \vee Fx)$    |
| b) $\forall x(Fx) \supset p$ | $\exists x(Fx \supset p)$ |