

# Philosophy 1630

## Extra Problems

Note: Many of these problems are taken from Warren Goldfarb's textbook *Deductive Logic*, which is what we use in Philosophy 0540.

### For Problem Set 1

1. The easiest way to do 1.1.10 is to formulate your definition so that you can prove the following: Every proper initial segment of any formula contains more left parentheses than right parentheses. So formulate your definition that way and prove this lemma.

### For Problem Set 2

2. Use truth-tables to determine the validity of the following formulae:

$$(p \vee q \rightarrow r) \rightarrow (p \rightarrow r)$$

$$(p \wedge q \rightarrow r) \rightarrow (p \rightarrow r)$$

3. Use truth-tables to determine whether the first formula implies the second:

$$\begin{array}{ll} p \vee r \equiv q \vee r & p \equiv q \\ (p \wedge \neg q) \vee (\neg q \wedge r) & (p \equiv q) \equiv r \end{array}$$

### For Problem Set 4

4. For each of the following formulae, find a structure that makes it true and another that makes it false.

$$\begin{array}{l} \exists x \forall y (Fyx \rightarrow Fyy) \\ \forall x \forall y (Fxy \rightarrow \exists z (Fxz \wedge Fyz)) \\ \forall x [\forall y (Fyx \rightarrow Fxy) \rightarrow \forall y (Fxy \rightarrow Fyx)] \end{array}$$

### **For Problem Set 5**

5. Give careful proofs of five logical equivalences or consequences from the list on p. 99.

### **For Problem Set 6**

6. Give examples to show that Lemma 2.3.28, which you proved as exercise 2.3.65, cannot be improved. That is, show that there are onto homomorphisms that do not respect identity statements and that there are one-one homomorphisms that do not respect quantified statements.

### **For Problem Set 7**

7. Give a careful proof of Corollary 3.1.6.