Conditions Relating to the Model Existence Lemma

The Satisfaction Properties

S0 If $\Gamma \in \Sigma$ and $\Delta \subseteq \Gamma$, then $\Delta \in \Sigma$

S1 If $\Gamma \in \Sigma$, then for no A do we have $A \in \Gamma$ and $\neg A \in \Gamma$

S2 If $\Gamma \in \Sigma$ and $\neg\neg A \in \Gamma$, then $\Gamma \cup \{A\} \in \Sigma$

S3 If $\Gamma \in \Sigma$ and $(A \vee B) \in \Gamma$, then either $\Gamma \cup \{A\} \in \Sigma$ or $\Gamma \cup \{B\} \in \Sigma$

S4 If $\Gamma \in \Sigma$ and $\neg(A \vee B) \in \Gamma$, then $\Gamma \cup \{\neg A\} \in \Sigma$ and $\Gamma \cup \{\neg B\} \in \Sigma$

S5 If $\Gamma \in \Sigma$ and $\exists x B(x) \in \Gamma$, and if the constant c does not occur in Γ or $\exists x B(x)$, then $\Gamma \cup \{B(c)\} \in \Sigma$

S6 If $\Gamma \in \Sigma$ and $\neg\exists x B(x) \in \Gamma$, then for every closed term t $\Gamma \cup \{\neg B(t)\} \in \Sigma$

S7 If $\Gamma \in \Sigma$, then $\Gamma \cup \{t = t\} \in \Sigma$, for every closed term t

S8 If $\Gamma \in \Sigma$ and $B(s) \in \Sigma$ and $s = t \in \Sigma$, then $\Gamma \cup \{B(t)\} \in \Sigma$

The Closure Properties

C1 If $A \in \Gamma$, then $\neg A \notin \Gamma$

C2 If $\neg\neg A \in \Gamma$, then $A \in \Gamma$

C3 If $A \vee B \in \Gamma$, then either $A \in \Gamma$ or $B \in \Gamma$

C4 If $\neg(A \vee B) \in \Gamma$, then $\neg A \in \Gamma$ and $\neg B \in \Gamma$

C5 If $\exists x B(x) \in \Gamma$, then $B(t) \in \Gamma$ for some closed term t of $\mathcal{L}^+$

C6 If $\neg\exists x B(x) \in \Gamma$, then $\neg B(t) \in \Gamma$ for every closed term t of $\mathcal{L}^+$

C7 For every closed term t of $\mathcal{L}^+$, $t = t \in \Gamma$

C8 If $B(s) \in \Gamma$ and $s = t \in \Gamma$, then $B(t) \in \Gamma$

Lemma (13.3, Model Existence Lemma). *Let $\mathcal{L}$ be a language, and let $\mathcal{L}^+$ be the result of adding to $\mathcal{L}$ infinitely many new constants $d_0, d_1, \ldots$. Let Σ be any family of sets of $\mathcal{L}^+$ sentences that has the satisfaction properties. Then every set $\Gamma \in \Sigma$ is satisfiable and, indeed, has a model in which each element of the domain is the denotation of some closed term of $\mathcal{L}^+$.*

Lemma (13.5, Term Models Lemma). *Let Γ^* be a set of sentences of $\mathcal{L}^+$ with the closure properties. Then Γ^* is satisfiable and, indeed, there is a model of Γ^* in which every element of the domain is the denotation of some closed term of $\mathcal{L}^+$.*

Lemma (13.6, Closure Lemma). *Let $\mathcal{L}$ be a language, and let $\mathcal{L}^+$ be the result of adding to $\mathcal{L}$ infinitely many new constants $d_0, d_1, \ldots$. If Σ is a family of sets of $\mathcal{L}^+$ sentences that has the satisfaction properties, then every set $\Gamma \in \Sigma$ of $\mathcal{L}$-sentences can be extended to a set Γ^* that has the closure properties.*

Lemma (13.2, Finite Character Lemma). *Let Σ be a family of sets of sentences having the satisfaction properties. Let Σ^* be the family of all sets of sentences each of whose finite subsets is in Σ. Then Σ^* has the satisfaction properties, too.*