Philosophy 1880
Advanced Deductive Logic

Instructor: Richard Heck
Office: 206 Corliss-Brackett
Office hours: TBA

Email: rgheck@brown.edu
Web site: http://rgheck.frege.org/
Office phone:(401)863-3217

Course Structure and Requirements

The course will meet Monday, Wednesday, and Friday, at 10am, in Wilson 203. Class meetings will consist primarily of lectures.

The text for the course is *Computability and Logic*, by Boolos, Burgess, and Jeffrey. Please get a copy of the latest edition, the fifth. The fourth edition is available on Josiah. But please do not just download pirated copies of this book. The people who wrote this book worked very hard to do so, and it is not one of those textbooks that is exorbitantly priced.

As with any mathematical subject-matter, it is impossible to learn this material without doing a lot of exercises. The book contains many, and problem sets drawing upon these exercises will need to be submitted several times during the semester. Students are *encouraged* to work on the problems together. Submitted material should be a student's own work, however.

There will be a final examination during the final exam period.

Grading Policies

Final grades will be determined by a variety of factors.

- The first and most important factor is that ***all of the problem sets must be completed and submitted for marking***. I'll let you off once, if you miss one, but of course you will get no credit for that problem set. Failure to submit all (but one) of the problem sets will ***automatically*** lead to a grade of NC. It is, quite simply, impossible to learn this material without doing a lot of problems, and students should actually plan to do a lot more problems than are assigned. Please note that the requirement is that the problem sets should be "completed", and by that I mean that one has given them a proper effort. Simply turning in a piece of paper with a few random jottings does not count as completing a problem set.

- It is impossible to fail the class if you have given it a proper effort.
- The final exam will count for half the grade. The problem sets will count for the other half.

Problem sets are due *in class* on the day specified. I will not accept late problem sets. On the other hand, you will find that I am quite prepared to grant extensions, so long as they are requested in advance, that is, at least one day prior to the due-date. Extensions will not be granted after that time except in very unusual and unfortunate circumstances. Because I am so reasonable, exploitation of my reasonableness will be taken badly.

Prerequisites

Formally speaking, Philosophy 0540 or 1630 is a prerequisite for this course. But more strictly speaking, we will be presuming a familiarity with basic logical notation, with how it can be used to represent the 'logical forms' of ordinary English statements and of mathematical claims, and with basic facts about validity, implication, and formal deduction. So you should understand what something like $\forall x \exists y(Fxy)$ might mean, and how it would differ from $\exists y \forall x(Fxy)$. And you should understand what it means to say that the latter implies the former, but not conversely, and how this could be shown.

Less specifically, this course is very mathematical in content. Perhaps the most important thing a student will need to be successful is a solid understanding of what it is to *prove* something mathematically. No particular mathematical knowledge is presumed, but a familiarity with 'mathematical induction' will be very helpful.

Doing Problem Sets Electronically

You are welcome to do your problem sets by hand or on a computer. But if you are going to do the latter, then I would strongly recommend that you not use a traditional "word processor" to do so. They are simply not optimized for mathematics, and their output is awful. A much better option is LaTeX, and if you want to use LaTeX in an environment that feels a lot like a word processor, then you can use LyX, which is a free and open source program that is available at http://www.lyx.org/. I am one of the lead developers of LyX, and will be happy to answer questions about it.

Especially if you have any intention of ever doing serious technical writing, you should start using LaTeX, with or without LyX, sooner rather than later. In the sciences, especially, it is the standard tool. Many scientific journals do not accept submissions in any other form.

Syllabus

The following syllabus is approximate—and very much so. The syllabus on the course website will be kept up to date.

27 January	No Meeting
29 January	Introductory Meeting, Chapter 1
1 February	Chapter 2
Problem Set 1	Due 8 February
3–8 February	Chapter 3, Section 4.1
Problem Set 2	Due 15 February
10–15 February	Chapters 5–6
Problem Set 3	Due 22 February
17–24 February	Chapter 7, Section 8.1
Problem Set 4	Due 2 March
26 February–4 March	Chapters 9–10
Problem Set 5	Due 11 March
7–14 March	Section 11.1, Chapter 13
Problem Set 6	Due 21 March
16–25 March	Chapters 14–15
Problem Set 7	Due 6 April
28 March–1 April	*No Class: Spring Break*
4–13 April	Chapter 16
Problem Set 8	Due 20 April
15–20 April	Chapter 17
Problem Set 9	Due 27 April
22–27 April	Chapter 18
29 April–9 May	Reading Period No classes scheduled, but it is likely we will go over somewhere and need to meet a couple times. Just so you know.
19 May, 2pm	Final Exam

Problem Sets

Note that in many cases you have a choice of what problems to do, and of course you're welcome to do more. You really *should* do more problems than these since that's really how you learn this kind of material.

In many cases, the easiest way to do one problem is to appeal to the result of a previous problem. You may do this, even if you did not do the previous problem, unless that is obviously ‘cheating’. (A good example would be to do 2.5 by appeal to 2.3 and 2.4, without doing either of those.) As the book notes, you should be *reading* all the problems that relate to material we have covered, anyway.

Problem sets are usually due a week after we finish the material in class.

Problem Set 1	1.2–1.5 (note that there is a typo in 1.5) It’s worth having a go at 1.7, as well. The interesting point about this problem is that doing it actually requires appeal to the axiom of choice, though informal proofs tend to mask this fact. 2.1–2.4; also, 2.10 and 2.11
Problem Set 2	3.1, 3.3, 3.5 4.1–4.3
Problem Set 3	5.1, 5.5, 5.10, 5.11 6.1, 6.3, 6.4, 6.7, 6.9 (using your coding from 6.7) If you want to challenge yourself, try 5.7–5.9 or 6.5
Problem Set 4	7.1, 7.3 (typo here), 7.4, 7.5, 7.6 For a challenge, try some of 7.12–7.17, which sketch a proof that the Ackerman function is not primitive recursive. Note that there is a typo in 7.15. 8.1, 8.2 For 8.1, what is wanted is a definition of a two-place function strt(x_1, x_2), similar to the definition of strt(x) on p. 90. The idea is to show that this function is primitive recursive. What is it that 8.2 implies? or at least starts to imply?
Problem Set 5	There are two options here, depending upon how strong your logic background is. If you have not previously taken logic, or you would like to review basic logic stuff, then you should do: 9.2–9.5 (typo in 9.3), 10.2–10.5 If you have previously taken logic, and you do not think you need to review basic logic stuff, then you should do: 9.4–9.7, 10.6, 10.9–10.11 For 9.4, you might want to reflect on the fact that, if we do not use enough parentheses, then the corresponding claim is not true. E.g., suppose we use no parentheses. Then $\neg A \vee B$ has as proper subformulas: A, B, $\neg A$, $A \vee B$. And both of these: $A, \neg A, B, \neg A \vee B$ and $A, B, A \vee B, \neg A \vee B$ are formation sequences for it. But neither contains all its subformulas. For the problems in Chapter 10, if you are asked to show that one thing A implies another thing B, then you need to show that every interpretation that makes A true makes B true. That is how implication is defined.

In 10.5 and 10.9, do not assume that Γ and Δ are finite! For 10.9(b), you can prove it in the following form: If $\Gamma \cup \{\neg\exists v_i B(v_i)\}$ is satisfiable, then $\Gamma \cup \{\neg B(v_i)\}$ is satisfiable. Note that satisfiability for *formulae* needs to be understood as truth under an interpretation *and an assignment*.

The statements of 10.10 ande 10.11 are somewhat confusing. The book asks you to show that the various statements hold for "equivalence over an interpretation". This notion is defined on p. 122, right after the completion of the proof. Since our treatment of quantification was somewhat different from the book's, this definition needs to be re-stated in terms of equivalence over an interpretation and an assignment. So two formulae F and G are equivalent over $\mathcal{M}$ and α just in case $\mathcal{M} \models_\alpha F$ iff $\mathcal{M} \models_\alpha G$. So what you want to prove is, e.g., for 10.10(d): If F and G are equivalent over $\mathcal{M}$ and α, then $\neg F$ and $\neg G$ are equivalent over $\mathcal{M}$ and α.

Yes, some of the parts of 10.10 are totally trivial; others are mostly trivial; as usual, the one that isn't trivial is the one involving quantification. For that one (f), you can just prove it in the following form: If for every β that is an i-variant of α, $F(v_i)$ is equivalent over $\mathcal{M}$ and β to $G(v_i)$, then $\forall v_i(F(v_i))$ is equivalent over $\mathcal{M}$ and α to $\forall v_i(G(v_i))$.

Note that, once you have done 10.10, you have all but done 10.11(a,b). In fact, you do not really have to do those problems. You could just explain why what I just said is true.

Problem Set 6

11.1–11.4

In the proof of Lemma 13.2 in the book, the cases of (S4)–(S8) were skipped. Do at least two of them.

You should also do cases (C4), (C6), and (C7) from the proof of the Closure Lemma that was presented in class. (There is a handout on the course website containing that proof.)

Note that there are only two ways, really, to do 11.3 and 11.4. The first, which is probably easiest, is to reason about interpretations: I.e., show that any interpretation that makes the premises true makes the conclusion true. An alternative would be to appeal to some of the facts about implication that were established in Examples 10.3 and 10.4. I'm not sure that is a very good method all by itself, but it could perhaps be mixed with the first

one in some interesting way.
For a challenge, do some of the sequence 13.8–13.13.
Note that there are typos in many of the unassigned problems in Ch 11. There is also a typo in 13.8: The definition of isomorphism is given in section 12.1 of the book, not in section 13.1. It is on page 139.

Problem Set 7

The problems for Chapter 14 are 14.1(b), 14.2, 14.3, and 14.6. However, since we have used a different sequent calculus from the one in the book, some of them need to be restated. As follows:
14.1(b): Show that Γ implies A iff some finite subset Γ_0 of Γ implies A.
14.3: Show that if $\Gamma, A, B \Rightarrow C$ is derivable, then $\Gamma, A \wedge B \Rightarrow C$ is derivable.
14.6: Show that if $\Gamma, A(t) \Rightarrow C$ is derivable, then so is $\Gamma, \forall x A(x) \Rightarrow C$.
The problems for Chapter 15 are 15.2, 15.5, 15.8, 15.10. But, again, since we have used a different notion of 'theory', they need to be restated. As follows:
15.2: Show that $\mathsf{Cl}(\Gamma)$, the closure of Γ, is closed, for any Γ. (Extra credit: Show that, if Δ is a closed theory and $\Gamma \subseteq \Delta$, then $\mathsf{Cl}(\Gamma) \subseteq \Delta$. I.e., $\mathsf{Cl}(\Gamma)$ is the smallest closed theory containing Γ.)
15.5: Let $\Gamma = \{A_1, A_2, \ldots\}$ be a countable set of sentences. Suppose that for no n is A_n derivable from $\{A_1, \ldots, A_{n-1}\}$. Show that there is no finite set Δ, whether it is a subset of Γ or not, that has the same theorems as Γ, i.e., such that $\mathsf{Cl}(\Gamma) = \mathsf{Cl}(\Delta)$.
For a challenge, try some of 15.8–15.10.

Problem Set 8

This is a longer one, as we're spending several sessions on Ch 16. Start early!
16.1, 16.3, 16.8–16.11, 16.17
Note: For 16.3, 16.8, and 16.9, you can consider the theory I called **R**. These are all true for it. And note that in 16.3, "correct" and "incorrect" just mean: true and false (in the standard interpretation).
For 16.10 and 16.11, the "recursion equations for addition" are (Q3) and (Q4) on p. 208.
For 16.17, you should use the theory the book calls **Q**. (I don't think the theory I called **Q** proves this result.)
Remember that what the book calls $\exists$-rudimentary sentences are what I called Σ_1-sentences, and what the book calls $\forall$-rudimentary sentences are what I called Π_1-sentences.

	For a challenge, try 16.4, the series 16.5–16.7, or the series 16.13–16.15
Problem Set 9	17.1, 17.4–17.6 Note that the point of 17.1 is that we can prove that no consistent extension of **Q** (or **R**) proves all true Π_1 sentences *without using diagonalization or any of the results of Ch. 17*, but *simply* by appealing to the existence of a set that is r.e. but not recursive (and what else we know about such sets, e.g., the results of Ch. 16). So you should *not* appeal to Gödel's Theorem in proving 17.1. For a challenge, try 17.2, 17.3 (generalize it further, if you can) and 17.7–17.8.